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Performance Analysis of Full Duplex on-regenerative Relay

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Abstract— **In this letter, non-regenerative Amplify-and-Forward (AF) relay systems based on half and full duplex schemes are investigated and their performance is analyzed and compared in terms of outage probability. Although the AF relay systems have been widely investigated in many previous literatures, most of them adopted a half duplex scheme due to hardware limitation and mathematical tractability. To the best of our knowledge, this letter is the first study to investigate the performance of the full duplex AF relay system considering practical hardware limitations. In full duplex AF relay systems, it is important to secure the isolation between transmit and receive antennas. Our numerical and simulation results show that there exists a threshold point of the isolation gain that the full duplex relay system outperforms the half duplex relay system.**

Index Terms— **Relay, amplify-and-forward, full duplex, half duplex.**

I. INTRODUCTION

DIVERSITY techniques using antenna arrays or RAKE receivers are conventional tools to combat fading in wireless communications. Recently, cooperative relaying has been recognized as an effective alternative to the conventional diversity techniques due to a significant gain through cooperation among nodes [1, 2]. Laneman *et al*. [2] proposed several cooperative diversity protocols and showed that they can achieve full diversity order.

Among the proposed protocols, non-regenerative Amplify-and-Forward (AF) scheme is the most practical and simplest since a relay node simply retransmits a received signal from a source node without decoding. Although the AF scheme has attracted much attention and has been widely investigated due to its practicality and simplicity in previous studies [3–7], most of them adopted a half-duplex scheme where a relay doesn't transmit and receive at the same time in the same resource. The half-duplex scheme can partially reduce the hardware cost because part of radio frequency components can be shared at receiving and transmitting and can easily prevent an oscillation phenomenon which is caused when the amplification gain of a relay node is larger than radio isolation between transmit and receive antennas. However, the half-duplex AF scheme's resource efficiency is seriously damaged because the orthogonality between reception and transmission should be guaranteed.

On the other hand, full duplex non-regenerative AF relays are widely deployed in code division multiple access (CDMA) and orthogonal frequency division multiple (OFDM)-based systems due to its practical feasibility [8]. A full duplex AF relay has a tradeoff between the resource utilization and power constraint. Compared to a half duplex AF relay, a full duplex AF relay has high resource utilization because it can simultaneously perform both transmission and reception without switching, while its transmit power can be seriously limited by an oscillation phenomenon which is caused when the amplification gain is larger than the isolation gain from transmitter to receiver. Thus, the performance of the full duplex AF relay is closely dependent on the isolation gain. Based on this motivation, we investigate the performance of the full duplex AF relay and compare it with that of the half duplex AF relay in terms of outage probability.

The rest of this letter is organized as follows: In Section II, full and half duplex AF relay systems are described and their outage probabilities are formulated. In Section III, the outage probabilities are asymptotically approximated. In Section IV, numerical results are shown. Finally, conclusions are drawn in Section V.

II. SYSTEM MODEL AND OUTAGE PROBABILITY

First, we consider a half duplex relay system which employs an AF scheme and consists of a source node, a relay node, and a destination node denoted by *s*, *d*, and *r*, respectively. In order to provide half duplex operation, we divide each time resource into two orthogonal slots whose durations are identical. In the first slot, a source node transmits its data to a relay node. In the second slot, the relay node amplifies and forwards the received message to a destination. For simplicity, we assume that the direct link between the source and destination nodes is blocked by intermediate obstacles as in [9]. We consider N channel uses, where N is sufficiently large. Then, the received signal at the destination node in a half duplex relay system can be represented by

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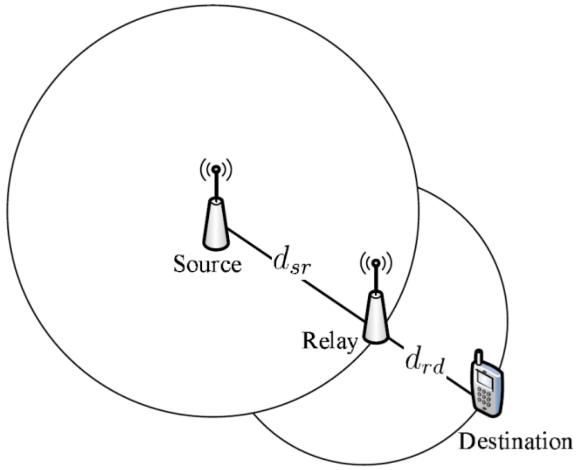

Fig. 1. AF relay system

$$y_H[n] = g_H h_{rd}(h_{sr}x[n] + n_r[n]) + n_d[n] \quad (1)$$

where n = $\frac{N}{2}$ + 1, ··· ,N, x[n] satisfying E[|x|2]= Ps is the signal transmitted by a source and is normally distributed $\sim \mathcal{N}(0, P_s)$, $h_{AB} \sim \mathcal{CN}(0, \sigma_{AB}^2)$ denotes the channel gain between the node A and B and remains constant during N channel uses, g_H is the amplification gain of a relay node, and $n_A[n] \sim \mathcal{CN}(0, N_0)$ denotes the additive white Gaussian noise (AWGN) at the node A. We analyze outage probability as a performance metric because it is a useful metric for block fading channels where the channel gain is random but remains constant during a code-block.

The mutual information and corresponding outage probability are given by [2]

$$I_H = \frac{1}{2}\log_2\left(1 + \frac{\frac{P_S}{N_0} g_H^2 |h_{sr}|^2 |h_{rd}|^2}{g_H^2 |h_{rd}|^2 + 1}\right)$$

$$P_H^{out} = \Pr[I_H < R] \quad (2)$$

where R is the required spectral efficiency, g_H is determined by the transmit power constraint of relay node and is given as [2]

$$g_H = \sqrt{\frac{P_r}{|h_{sr}|^2 P_S + N_0}} \quad (3)$$

because the amplification gain in a half duplex AF system should be limited by the relay node's maximal allowable transmit power P_r, and $|h_{AB}|^2$ is an exponentially distributed random variable. The mean value of $|h_{AB}|^2$, σ_{AB}^2 is given by $d_{AB}^{-\alpha}$ where d_{AB} is the distance from the node A and B and α is the path loss exponent.

Contrary to the half duplex relay, a full duplex relay node can simultaneously transmit its received data to a destination node while receiving data from a source node. In the full duplex relay, it is very important to obtain a sufficient electrical isolation gain between the transmit and receive circuitry because we should control the amplification gain to be less than the isolation gain to prevent an oscillation. The received signal at the destination node in the full duplex AF system can be represented by

$$y_F[n] = g_F h_{rd}\left(h_{sr}\frac{x[n]}{\sqrt{2}} + n_r[n]\right) + n_d[n] \quad (4)$$

where n = 1, · · · ,N and the amplification gain g_F should be limited by both allowable transmit power and maximum allowable amplification gain of a relay node. Thus, gF can be obtained as

$$\begin{aligned} g_F &= \min\left(\sqrt{\frac{\frac{P_r}{2}}{|h_{sr}|^2 \frac{P_s}{2} + N_0}}, g_{max}\right) \\ &= \min\left(\sqrt{\frac{\frac{P_r}{2}}{|h_{sr}|^2 \frac{P_s}{2} + N_0}}, G_I\right) \\ &= \begin{cases} \sqrt{\frac{\frac{P_r}{2}}{|h_{sr}|^2 \frac{P_s}{2} + N_0}}, & |h_{sr}|^2 \geq \frac{\frac{P_r}{G_I^2} - 2N_O}{P_S} \\ G_I & otherwise \end{cases} \end{aligned} \quad (5)$$

where transmit powers of source and relay nodes are reduced by half for fair performance comparison with the half duplex scheme because the full duplex scheme's total transmit power is two times as much as that of the half duplex scheme, and maximum allowable amplification gain g_{max} of the full duplex relay should be determined as an electrical isolation gain of a relay node, G_I, to prevent an oscillation of the relay node. G_I is defined as the ratio of the transmit signal power to the signal power to be fed back from a transmit antenna to a receive antenna.

It should be noted that although we ignored the direct link between source and destination nodes for simplicity, the direct signal from the source node can be easily combined with that from a relay node at the destination in CDMA and OFDM systems through RAKE receivers and cyclic prefix, respectively. The mutual information and corresponding outage probability of the full duplex relay system are given by [2]

$$I_F = \log\left(1 + \frac{\frac{P_s}{2N_0} g_F^2 |h_{sr}|^2 |h_{rd}|^2}{g_F^2 |h_{rd}|^2 + 1}\right)$$

$$P_H^{out} = \Pr[I_F < R] \quad (6)$$

III. ASYMPTOTIC ANALYSIS

Although outage probabilities of both the half and full duplex relay systems are formulated in Eqs. (2) and (6), their closed-form solutions cannot be obtained because it is well known that their statistical distributions cannot be easily obtained [2]. Thus, we rely on an asymptotic analysis to capture the effect of key parameters. We assume that $P_s = P_r = P$ and P is asymptotically large. In addition, it is also assumed that G_I is sufficiently large. The validity of the assumption will be justified in the following Sect. IV. Then, the effective amplification gain and the corresponding outage probability of the full duplex relay system can be approximated as

$$g_F \approx \sqrt{\frac{\frac{P_r}{2}}{|h_{sr}|^2 \frac{P_s}{2} + N_0}} \approx \frac{1}{|h_{sr}|}$$

$$P_F^{out} \approx \Pr\left[\frac{2|h_{sr}|^2 |h_{rd}|^2}{|h_{sr}|^2 + |h_{rd}|^2} < \frac{4(2^R - 1)}{\rho}\right] \quad (7)$$

Where ρ is defined to be $\frac{P}{N_0}$ and $\frac{2|h_{sr}|^2|h_{rd}|^2}{|h_{sr}|^2+|h_{rd}|^2}$ is the harmonic mean of two exponentially distributed random variables $|h_{sr}|^2$ and $|h_{rd}|^2$, whose the closed-form distribution can be obtained in [10]. Thus, $P_{\mathrm{F}}^{\mathrm{out}}$ in Eq. (7) can be rewritten as

$$P_F^{out} \approx 1 - \frac{R'}{\sqrt{d_{sr}^{-\alpha} d_{rd}^{-\alpha}}} e^{\frac{R'(d_{sr}^{-\alpha} + d_{rd}^{-\alpha})}{2 d_{sr}^{-\alpha} d_{rd}^{-\alpha}}} K_1\left(\frac{R'}{\sqrt{d_{sr}^{-\alpha} d_{rd}^{-\alpha}}}\right) \quad (8)$$

Where R` denotes and $K_1(\cdot)$ is the first-order modified Bessel function of the second R kind. Since $e^{-x} \approx 1 - x$ and $K_1(x) \approx \frac{1}{x}$ when x is asymptotically small, Eq. (8) can be approximated again as

$$P_F^{out} \approx \frac{2(2^R - 1)(d_{sr}^{-\alpha} + d_{rd}^{-\alpha})}{\rho d_{sr}^{-\alpha} d_{rd}^{-\alpha}} \quad (9)$$

Similarly, the outage probability of the half duplex relay system, $P_{\mathrm{H}}^{\mathrm{out}}$, can be also approximated as

$$P_H^{out} \approx \frac{2(2^R - 1)(d_{sr}^{-\alpha} + d_{rd}^{-\alpha})}{\rho d_{sr}^{-\alpha} d_{rd}^{-\alpha}} \quad (9)$$

If we set the spectral efficiency $R = r \log_2 \rho$ where denotes a multiplex gain, diversity-multiplexing tradeoff which is defined as $d \triangleq -\lim_{\rho\to\infty} \frac{\log_2 P^{\mathrm{out}}}{\log_2 \rho}$ [11] can be obtained as

$$d_{\mathrm{F}} = 1 - r, \quad d_{\mathrm{H}} = 1 - 2r, \quad (11)$$

which clearly shows that the degree of freedom of the full duplex scheme is 1 while that of the half duplex scheme is $\frac{1}{2}$ and diversity gain of both schemes is 1 because we ignored the direct link from a source node to a destination node.

IV. NUMERICAL RESULTS

In this section, numerical results are shown when R = 0.5 bps/Hz, $d_{sr} = d_{rd}$ = 1000 m, α = 4, and $N_0 = -104$ dBm where the noise power spectral density is −174 dBm/Hz and channel bandwidth is 10 MHz. In addition, Monte-Carlo simulation results are also presented to verify the numerical results. Our simulation is based on Monte-Carlo method. All channel gains are randomly and iteratively generated according to their distributions and configurations. Then, we count outage events defined in Eqs. (2) and (6) to obtain outage probability.

Fig. 2 shows outage probability for varying transmit power when G_I = 80 dB. It is shown that the full duplex scheme outperforms the half duplex scheme regardless of the transmit power level and our approximations agrees well with the exact simulation results as the transmit power increases. Fig. 3 shows outage probability for varying isolation gain when the transmit power P is 43 dBm which is real transmit power of base station and relay in current mobile communication systems. It is shown that the outage probability of the full duplex relay system is higher than that of the half duplex system if sufficient isolation is not secured, that is $G_I < 65$. On the other hand, the performance of the full duplex system outperforms that of the half duplex system if G_I is larger than 65 dB. It can be also demonstrated that our assumption and approximation is valid when $G_I > 70$ dB. In addition, when transmit power is sufficiently large and isolation gain can be sufficiently obtained, the full duplex system always outperforms the half duplex system, which can be shown from Eqs. (9) and (10) as follows:

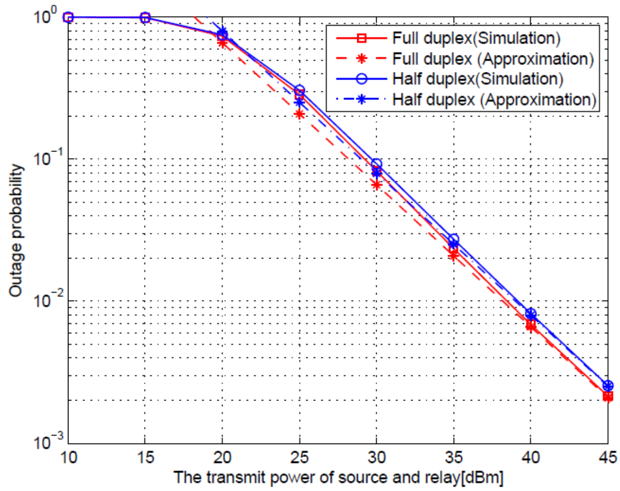

Fig. 2. Outage probability vs. transmit power when G_I = 80dB.

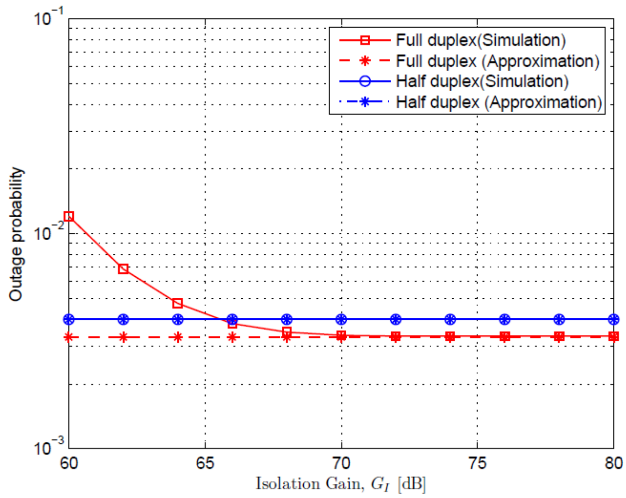

Fig. 3. Outage probability vs. isolation gain when P = 43dBm

$$\frac{P_H^{out}}{P_F^{out}} = \frac{2^R+1}{2} \geq 1 \quad (12)$$

V. CONCLUSIONS

In this letter, we investigated AF relay systems based on full and half duplex schemes. Although the half duplex AF relay systems have been widely investigated, the degree of freedom of resource is reduced because the resource should be divided into two orthogonal parts. On the other hand, the full duplex AF relays do not damage the degree of freedom while their outage probability can be degraded because their effective transmit power is regulated by the electrical isolation gain between their transmit and receive antennas. Our simulation and numerical results clearly showed that the full duplex relay system outperforms the half duplex system if sufficient isolation gain can be obtained.

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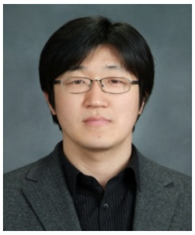

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